Interrogation de mathématique - 4

(Chapitre 2)

1) Simplifier les expressions suivantes :

a) $\sqrt{\sqrt{2}} \cdot \sqrt{\sqrt{2}}$ b) $\sqrt{6^2 + 8^2}$

c) $\sqrt{x^4}$ d) $\sqrt{9x^2 - 30x + 25}$

e) $\sqrt{x^2}$ f) $\sqrt{x^3}$

2) Rationnaliser les dénominateurs des fractions suivantes :

a) $\frac{\sqrt{21}}{\sqrt{7}}$ b) $\frac{7}{\sqrt{11} - 2}$ c) $\frac{6}{\sqrt{7} - \sqrt{5}}$ d) $\frac{2}{\sqrt{2} + \sqrt{7} - \sqrt{5}}$

3) Effectuer les opérations suivantes :

a) $\left(\sqrt{3} - \sqrt{2}\right)^2$ b) $\left(\sqrt{12} + 3\sqrt{3}\right)^2$

c) $\left(\sqrt{x} - 3\right)^2$ d) $\left(3\sqrt{11} + \sqrt{5}\right) \cdot \left(3\sqrt{11} - \sqrt{5}\right)$

e) $\frac{1}{\sqrt{3} + 1} + \frac{1}{\sqrt{3} - 1}$

1) a) $\sqrt{\sqrt{2}\cdot\sqrt{2}} = \sqrt{\sqrt{2}\cdot\sqrt{2}} = \sqrt{2}$

b) $\sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10$

c) $\sqrt{x^4} = \sqrt{(x^2)^2} = |x^2| = x^2$ car $x^2 \geq 0$

d) $\sqrt{9x^2-30x+25} = \sqrt{(3x-5)^2} = |3x-5|$

e) $\sqrt{x^2} = |x|$

f) $\sqrt{x^3} = \sqrt{x^2\cdot x} = \sqrt{x^2}\cdot\sqrt{x} = |x|\sqrt{x} = x\cdot\sqrt{x}$ car $x \geq 0$ ici

2) a) $\frac{\sqrt{21}}{\sqrt{7}} = \sqrt{\frac{21}{7}} = \sqrt{3}$

b) $\frac{7}{\sqrt{11}-2} = \frac{7}{\sqrt{11}-2}\cdot\frac{\sqrt{11}+2}{\sqrt{11}+2} = \frac{7(\sqrt{11}+2)}{11-4} = \frac{7\cdot(\sqrt{11}+2)}{7} = \sqrt{11}+2$

c) $\frac{6}{\sqrt{7}-\sqrt{5}} = \frac{6}{\sqrt{7}-\sqrt{5}}\cdot\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{6(\sqrt{7}+\sqrt{5})}{7-5} = \frac{3\cdot 2(\sqrt{7}+\sqrt{5})}{2} = 3(\sqrt{7}+\sqrt{5})$

d) $\frac{2}{\sqrt{2}+\sqrt{7}-\sqrt{5}} = \frac{2}{(\sqrt{2}+\sqrt{7})-\sqrt{5}}\cdot\frac{(\sqrt{2}+\sqrt{7})+\sqrt{5}}{(\sqrt{2}+\sqrt{7})+\sqrt{5}} = \frac{2(\sqrt{2}+\sqrt{7}+\sqrt{5})}{(\sqrt{2}+\sqrt{7})^2-(\sqrt{5})^2}$

$= \frac{2(\sqrt{2}+\sqrt{7}+\sqrt{5})}{2+2\sqrt{14}+7-5} = \frac{2(\sqrt{2}+\sqrt{7}+\sqrt{5})}{4+2\sqrt{14}} = \frac{2(\sqrt{2}+\sqrt{7}+\sqrt{5})}{2(2+\sqrt{14})}\cdot\frac{2-\sqrt{14}}{2-\sqrt{14}}$

$= \frac{(\sqrt{2}+\sqrt{7}+\sqrt{5})\cdot(2-\sqrt{14})}{4-14} = \frac{(\sqrt{2}+\sqrt{7}+\sqrt{5})(2-\sqrt{14})}{-10}$

3) a) $(\sqrt{3}-\sqrt{2})^2 = 3-2\sqrt{6}+2 = 5-2\sqrt{6}$

b) $(\sqrt{12}+3\sqrt{3})^2 = (2\sqrt{3}+3\sqrt{3})^2 = (5\sqrt{3})^2 = 5^2\cdot(\sqrt{3})^2 = 25\cdot 3 = 75$

c) $(\sqrt{x}-3)^2 = x-6\sqrt{x}+9$

d) $(3\sqrt{11}+\sqrt{5})(3\sqrt{11}-\sqrt{5}) = (3\sqrt{11})^2-(\sqrt{5})^2 = 9\cdot 11-5 = 99-5 = 94$

e) $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{3}-1} = \frac{1}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1}+\frac{1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}$

$= \frac{\sqrt{3}-1+\sqrt{3}+1}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

ou $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}-1+\sqrt{3}+1}{(\sqrt{3}+1)\cdot(\sqrt{3}-1)} = \frac{2\sqrt{3}}{3-1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$