

Interrogation de mathématique

(Chapitre 1 - 1)

Résoudre par factorisation les équations $f(x) = 0$ suivantes si :

- 1) $f(x) = (x-3)^2 - 121$
- 2) $f(x) = 6x^2 - 13x + 6$
- 3) $f(x) = 12x^3 - 30x^2 + 18x$
- 4) $f(x) = 3x^2 - 21x - 132$
- 5) $f(x) = (x-1)(x^2-4) - (x-1)(x-3) + (x-1)$
- 6) $f(x) = x^3 + 2x^2 - x - 2$

$$1) f(x) = (x-3)^2 - 121 = 0 \text{ et } x \in \mathbb{R}$$

$$\Leftrightarrow x^2 - 6x + 9 - 121 = 0$$

$$\Leftrightarrow x^2 - 6x - 112 = 0$$

$$\Leftrightarrow (x+8)(x-14) = 0$$

$$\Leftrightarrow x \in \{-8; 14\}$$

$$\begin{cases} m+n = -6 \\ m \cdot n = -112 \end{cases} \Leftrightarrow \begin{cases} m = +8 \\ n = -14 \end{cases}$$

ou $(x-3)^2 - 121 = 0 \text{ et } x \in \mathbb{R}$

$$\Leftrightarrow (x-3)^2 - 11^2 = 0$$

$$\Leftrightarrow (x-3-11)(x-3+11) = 0$$

$$\Leftrightarrow (x-14)(x+8) = 0 \Leftrightarrow x \in \{-8; 14\}$$

$$2) f(x) = 6x^2 - 13x + 6 = 0 \text{ et } x \in \mathbb{R} \begin{cases} m+n = -13 \\ m \cdot n = 36 \end{cases} \Leftrightarrow \begin{cases} m = -9 \\ n = -4 \end{cases}$$

$$\Leftrightarrow 6x^2 - 9x - 4x + 6 = 0$$

$$\Leftrightarrow 3x(2x-3) - 2(2x-3) = 0$$

$$\Leftrightarrow (2x-3)(3x-2) = 0 \Leftrightarrow x \in \left\{ \frac{3}{2}; \frac{2}{3} \right\}$$

$$3) f(x) = 12x^3 - 30x^2 + 18x = 0 \text{ et } x \in \mathbb{R}$$

$$\Leftrightarrow 6x^2(3x^2 + 2x - 5) = 0$$

$$\Leftrightarrow 6x^2[3x^2 + 5x - 3x - 5] = 0$$

$$\Leftrightarrow 6x^2[x(3x+5) - 1 \cdot (3x+5)] = 0$$

$$\Leftrightarrow 6x^2(3x+5)(x-1) = 0 \Leftrightarrow x \in \left\{ 0; -\frac{5}{3}; +1 \right\}$$

$$\begin{cases} m+n = +2 \\ m \cdot n = -15 \end{cases} \Leftrightarrow \begin{cases} m = +5 \\ n = -3 \end{cases}$$

$$4) f(x) = 3x^2 - 21x - 132 = 0 \text{ et } x \in \mathbb{R}$$

$$\Leftrightarrow 3[x^2 - 7x - 44] = 0$$

$$\Leftrightarrow (x-11)(x+4) = 0$$

$$\Leftrightarrow x \in \{11; -4\}$$

$$\begin{cases} m+n = -7 \\ m \cdot n = -44 \end{cases}$$

$$\Leftrightarrow \begin{cases} m = -11 \\ n = +4 \end{cases}$$

$$5) f(x) = (x-1)(x^2-4) - (x-1)(x-3) + (x-1) = 0 \text{ et } x \in \mathbb{R}$$

$$\Leftrightarrow (x-1) [(x^2-4) - (x-3) + 1] = 0$$

$$\Leftrightarrow (x-1) (x^2 - x) = 0$$

$$\Leftrightarrow (x-1) \cdot x \cdot (x-1) = 0 \Leftrightarrow x \in \{1; 0\}$$

$$6) f(x) = x^3 + 2x^2 - x - 2 = 0 \text{ et } x \in \mathbb{R}$$

$$\Leftrightarrow (x-1)(x^2 + 3x + 2) = 0$$

$$\Leftrightarrow (x-1)(x+1)(x+2) = 0$$

$$\Leftrightarrow x \in \{1; -1; -2\}$$

ou

$$f(x) = x^3 + 2x^2 - x - 2 = 0$$

$$\Leftrightarrow (x^3 + 2x^2) - (x+2) = 0$$

$$\Leftrightarrow x^2(x+2) - 1 \cdot (x+2) = 0$$

$$\Leftrightarrow (x+2)(x^2 - 1) = 0$$

$$\Leftrightarrow (x+2)(x-1)(x+1) = 0 \Leftrightarrow x \in \{-2; 1; -1\}$$

$$f(1) = 1 + 2 - 1 - 2 = 0$$

$$\begin{array}{c|ccc|c} 1 & 2 & -1 & -2 \\ & 1 & 3 & 2 \\ \hline 1 & 1 & 3 & 2 & 0 \end{array}$$

$$\begin{cases} m+n=3 \\ m \cdot n=2 \end{cases}$$

$$\Leftrightarrow \begin{cases} m=2 \\ n=1 \end{cases}$$