Exercices

9 ***Résoudre dans*** $\mathbb{R}^2$ avec la méthode de Cramer :

1 $\begin{cases} x + y = 7 \\ x - y = 9 \end{cases}$

2 $\begin{cases} \frac{x}{2} + y = 2 \\ x + \frac{y}{3} = 1 \end{cases}$

3 $\begin{cases} 2x + 5y = 1 \\ 5x + 2y = 1 \end{cases}$

4 $\begin{cases} x + 3 = y - 1 \\ -3x + 3y = 12 \end{cases}$

5 $\begin{cases} y = 3x + 4 \\ y = -2x + 1 \end{cases}$

6 $\begin{cases} \frac{x+y}{2} = \frac{1}{4} \\ \frac{x-y}{3} = 1 \end{cases}$

7 $\begin{cases} 2x = 3y + 5 \\ 4y = 3x - 1 \end{cases}$

8 $\begin{cases} \frac{x}{3} + \frac{y}{2} = 1 \\ 2x + 3y = 7 \end{cases}$

9 $\begin{cases} \frac{x}{5} - y = 3 \\ -x + 5y = 15 \end{cases}$

exercice 9 : (3)

$$\begin{cases} 2x + 5y = 1 \\ 5x + 2y = 1 \end{cases} \quad \text{et } (x,y) \in \mathbb{R}^2 \quad (\text{ou } \mathbb{R} \times \mathbb{R})$$

$\Leftrightarrow D = \begin{vmatrix} 2 & 5 \\ 5 & 2 \end{vmatrix} = 2 \cdot 2 - 5 \cdot 5 = 4 - 25 = -21 \neq 0$ donc le système admet un unique couple $(x;y)$ solution

$$D_x = \begin{vmatrix} 1 & 5 \\ 1 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 5 = -3$$

$$D_y = \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} = 2 \cdot 1 - 5 \cdot 1 = -3$$

et $x = \frac{D_x}{D} = \frac{-3}{-21} = \frac{1}{7}$ et $y = \frac{D_y}{D} = \frac{-3}{-21} = \frac{1}{7}$

$\Leftrightarrow (x;y) \in \left\{ \left(\frac{1}{7} ; \frac{1}{7} \right) \right\}$

exercice 9 : (6)

$$\begin{cases} \frac{x+y}{2} = \frac{1}{4} \\ \frac{x-y}{3} = 1 \end{cases} \text{ et } (x;y) \in \mathbb{R}^2 \iff \begin{cases} 2x+2y = 1 \\ x-y = 3 \end{cases}$$

$\iff D = \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} = 2 \cdot (-1) - 1 \cdot 2 = -4 \neq 0$ donc un unique couple solution

$D_x = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -1 - 6 = -7$ et $D_y = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$

et $x = \frac{D_x}{D} = \frac{-7}{-4} = \frac{7}{4}$ et $y = \frac{D_y}{D} = \frac{5}{-4} = \frac{-5}{4}$

$\iff (x;y) \in \left\{ \left(\frac{7}{4} ; \frac{-5}{4} \right) \right\}$

7 $\begin{cases} 2x = 3y + 5 \\ 4y = 3x - 1 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} 2x - 3y = 5 \\ -3x + 4y = -1 \end{cases}$

$\Leftrightarrow \begin{cases} D = \begin{vmatrix} 2 & -3 \\ -3 & 4 \end{vmatrix} = -1 \neq 0 \\ D_x = \begin{vmatrix} 5 & -3 \\ -1 & 4 \end{vmatrix} = 17 \\ D_y = \begin{vmatrix} 2 & 5 \\ -3 & -1 \end{vmatrix} = 13 \end{cases}$

et $(x;y) \in \{(-17;-13)\}$