$\sin(x+207°) + \cos(2x-13°) = 0$ et $x \in \mathbb{R}$

$\Leftrightarrow \sin(x+207°) = -\cos(2x-13°)$

$\Leftrightarrow \sin(x+207°) = -\sin\left(\cancel{\frac{\pi}{2}}^{90°} - (2x-13°)\right)$

$\Leftrightarrow \sin(x+207°) = -\sin(103° - 2x)$

$\Leftrightarrow \sin(x+207°) = \sin(2x - 103°)$

$\Leftrightarrow \begin{cases} x+207° = 2x - 103° + k360° \\ \text{ou} \\ x+207° = 180° - (2x-103°) + k360° \end{cases}$

$\Leftrightarrow \begin{cases} -x = -310° + k360° \\ \text{ou} \\ 3x = 76° + k360° \end{cases}$

$$\begin{array}{r} 283 \\ -\ 207 \\ \hline 76 \end{array}$$

$\Leftrightarrow \begin{cases} x = 310° + k360° \\ \text{ou} \\ x = \frac{76}{3}° + k\frac{360°}{3} \end{cases}$

$\Leftrightarrow x \in \left\{ 310° + k360° \,;\, \frac{76}{3}° + k120° \right\}$

2 B - C sciences Vendredi 1 mai 2020

Examen préparatoire de trigonométrie

1) Compléter les cases **non grisées** du tableau suivant :

x	$37\pi/6$	$-5\pi/2$	$7\pi/4$	$15\pi/3$	$-13\pi/6$
cos(x)	$\frac{\sqrt{3}}{2}$		$\frac{\sqrt{2}}{2}$		$\frac{\sqrt{3}}{2}$
sin(x)		-1		0	
tan(x)		X		0	
cot(x)	$\sqrt{3}$		-1		$-\sqrt{3}$

2) Soit $\tan(x) = \frac{-5}{12}$, alors

$\cot(x) = \frac{1}{\tan(x)} = \frac{-12}{5}$

et

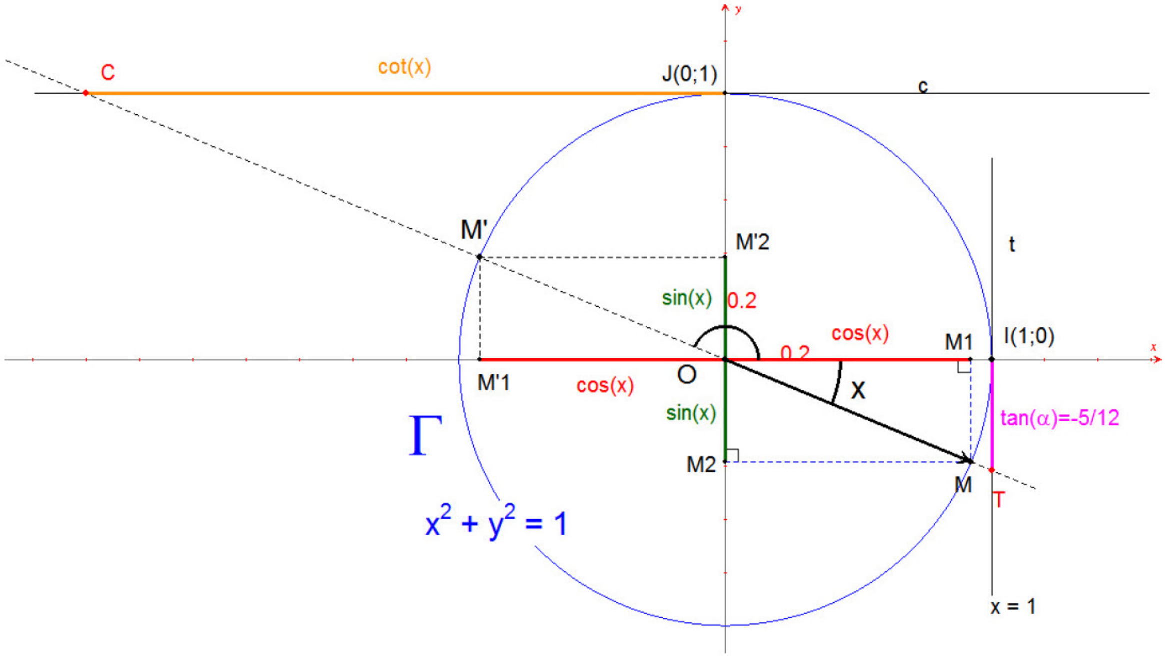
$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{-5}{12}$ et $\sin^2(x) + \cos^2(x) = 1$

$\Leftrightarrow \sin(x) = \frac{-5}{12}\cos(x)$ et $\left(\frac{-5}{12}\cos(x)\right)^2 + \cos^2(x) = 1$

et $\cos^2(x)\left(\frac{25}{144}+1\right) = 1$

et $\cos^2(x) = \frac{144}{169}$

$$\Leftrightarrow \begin{cases} \cos(x) = \frac{12}{13} \text{ et } \sin(x) = \frac{-5}{12}\cdot\frac{12}{13} = \frac{-5}{13} \\ \text{ou} \\ \cos(x) = \frac{-12}{13} \text{ et } \sin(x) = \frac{5}{13} \end{cases}$$



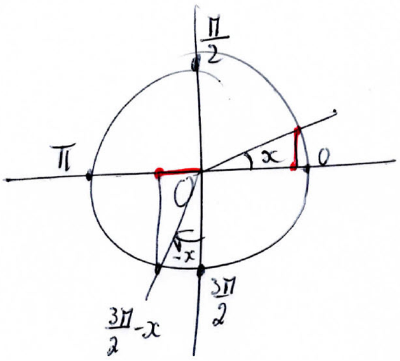
C
cot(x)
J(0;1)
c
M'
M'2
t
sin(x)
0.2
cos(x)
M1
I(1;0)
0.2
M'1
cos(x)
O
x
sin(x)
tan(α)=-5/12
Γ
M2
M
T
x² + y² = 1
x = 1

exe 3: $\cos\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{3\pi}{2}-x\right)-\cos\left(\frac{5\pi}{2}-x\right)+\cos\left(\frac{7\pi}{2}-x\right)=$

$\cancel{\cos\left(\frac{\pi}{2}-x\right)}+\cos\left(\frac{3\pi}{2}-x\right)-\cancel{\cos\left(\frac{\pi}{2}-x\right)}+\cos\left(\frac{3\pi}{2}-x\right)=$

$2\cos\left(\frac{3\pi}{2}-x\right)=$

$-2\sin(x)$



exe 4 : a) $\cos(x) = \frac{-\sqrt{2}}{2}$ et $x \in \mathbb{R}$

$$\Leftrightarrow \cos(x) = -\cos\left(\frac{3\pi}{4}\right) \Leftrightarrow \begin{cases} x = \frac{3\pi}{4} + k2\pi \\ \text{ou} \\ x = -\frac{3\pi}{4} + k2\pi \end{cases}$$

$$\Leftrightarrow x \in \left\{ \frac{3\pi}{4} + k2\pi \; ; \; \frac{-3\pi}{4} + k2\pi \right\}$$

b) $\sin(x) = \cos(2x)$ et $x \in \mathbb{R}$

$\Leftrightarrow \cos\left(\frac{\pi}{2} - x\right) = \cos(2x)$

$\Leftrightarrow \begin{cases} \frac{\pi}{2} - x = 2x + k2\pi \\ \text{ou} \\ \frac{\pi}{2} - x = -2x + k2\pi \end{cases} \Leftrightarrow \begin{cases} -3x = \frac{-\pi}{2} + k2\pi \\ \text{ou} \\ x = -\frac{\pi}{2} + k2\pi \end{cases}$

$\Leftrightarrow x = \frac{\pi}{6} + k\frac{2\pi}{3}$ ou $x = \frac{-\pi}{2} + k2\pi$

$\Leftrightarrow x \in \left\{ \frac{\pi}{6} + k\frac{2\pi}{3} ; \frac{-\pi}{2} + k2\pi \right\}$

c) $\tan^2(t) - 4\tan(t) + 3 = 0$ et $t \in \mathbb{R} - \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$

$\Leftrightarrow$ $y = \tan(t)$ et $y^2 - 4y + 3 = 0$

et $(y-3)(y-1) = 0$

$\Leftrightarrow$ $\tan(t) = 3$ ou $\tan(t) = 1$ avec le tableau

$\Leftrightarrow$ $\tan(t) \cong \tan(1{,}25)$ calcul. ou $\tan(t) = \tan(\frac{\pi}{4})$

$\Leftrightarrow$ $t \in \{1{,}25 + k\pi \,;\, \frac{\pi}{4} + k\pi\}$

d) $15\cos^2(t) + 2\cos(t) - 8 = 0$ et $t \in \mathbb{R}$

$\Leftrightarrow \cos(t) = y$ et $15y^2 + 2y - 8 = 0$

et $15y^2 + 12y - 10y - 8 = 0$

et $3y(5y+4) - 2(5y+4) = 0$

et $(5y+4)(3y-2) = 0$

$\begin{cases} m+n = 2 \\ m \cdot n = 15 \cdot (-8) \\ \quad = 3 \cdot 5 \cdot 2 \cdot 2 \cdot 2 (-1) \end{cases}$

$\Leftrightarrow \begin{cases} m = 3 \cdot 2 \cdot 2 = 12 \\ n = -5 \cdot 2 = -10 \end{cases}$

$\Leftrightarrow \cos(t) = \frac{-4}{5}$ ou $\cos(t) = \frac{2}{3}$

$\Leftrightarrow \cos(t) \cong \cos(2{,}50)$ ou $\cos(t) \cong \cos(0{,}84)$

$\Leftrightarrow \begin{cases} t \cong 2{,}5 + k2\pi \\ \text{ou} \\ t \cong -2{,}5 + k2\pi \end{cases}$ ou $\begin{cases} t \cong 0{,}84 + k2\pi \\ \text{ou} \\ t \cong -0{,}84 + k2\pi \end{cases}$

$\Leftrightarrow t \in \{2{,}5 + k2\pi \,;\, -2{,}5 + k2\pi \,;\, 0{,}84 + k2\pi \,;\, -0{,}84 + k2\pi\}$

Résoudre l'équation $2\cos(\alpha) + \sin(\alpha) = -2$ et $\alpha \in \mathbb{R}$

$\Leftrightarrow \quad x = \cos(\alpha)$ et $y = \sin(\alpha)$

et $2x + y = -2$ (équation d'une droite d)

$\Leftrightarrow \quad x = \cos(\alpha)$ et $y = \sin(\alpha)$

et $\begin{cases} 2x + y = -2 \\ \text{et} \\ x^2 + y^2 = 1 \end{cases}$ $*$

$\ast$ $\begin{cases} 2x + y = -2 \\ \text{et} \\ x^2 + y^2 = 1 \end{cases} \iff \begin{cases} y = -2x - 2 \\ x^2 + (+2x+2)^2 = 1 \end{cases}$

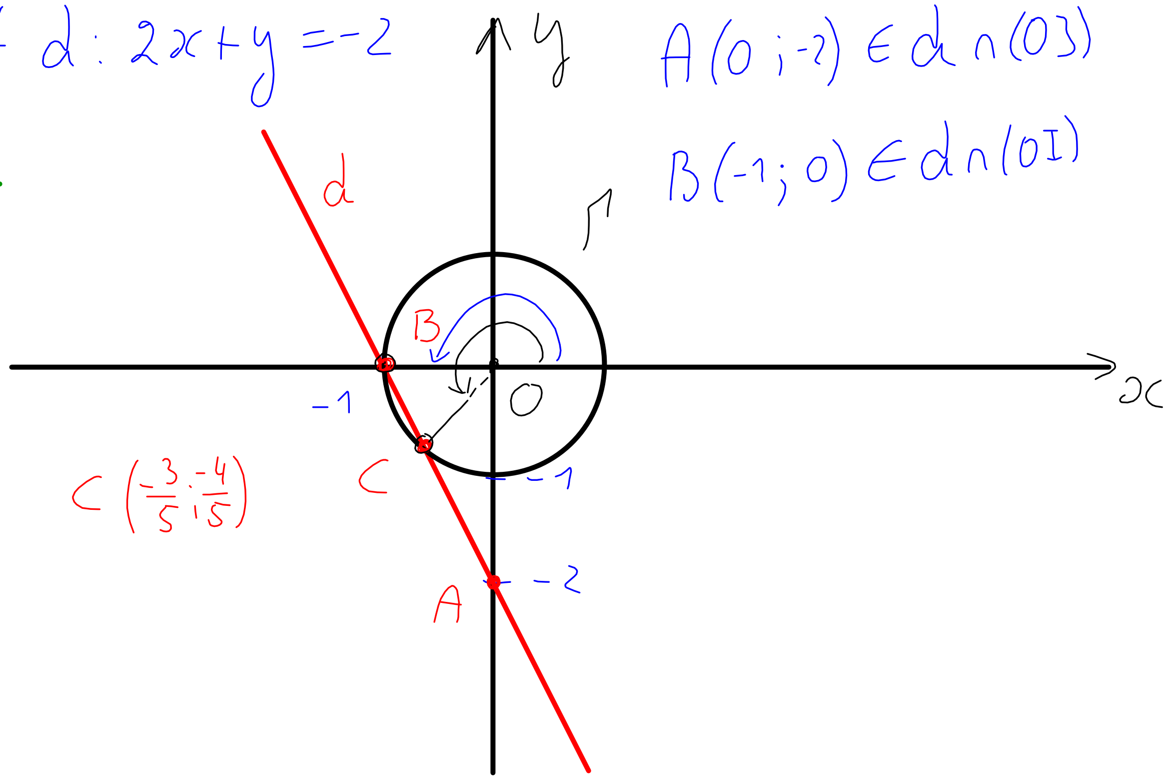
$\iff \begin{cases} y = -2x - 2 \\ 5x^2 + 8x + 3 = 0 \end{cases}$

$\iff \begin{cases} y = -2x - 2 \\ (x+1)(5x+3) = 0 \end{cases} \iff (x;y) \in \left\{ (-1;0) ; \left(\frac{-3}{5} ; \frac{-4}{5}\right) \right\}$

Soit $d : 2x + y = -2$

$A(0\,;-2) \in d \cap (OJ)$

$B(-1\,;0) \in d \cap (OI)$



$C\left(\frac{-3}{5} ; \frac{-4}{5}\right)$

$$\iff \begin{cases} x = \cos(\alpha) = -1 \text{ et } y = \sin(\alpha) = 0 \\ \text{ou} \\ x = \cos(\alpha) = \frac{-3}{5} \text{ et } y = \sin(\alpha) = \frac{-4}{5} \end{cases}$$

$$\iff \cos(\alpha) = \cos(\pi) \text{ et } \sin(\alpha) = \sin(0)$$

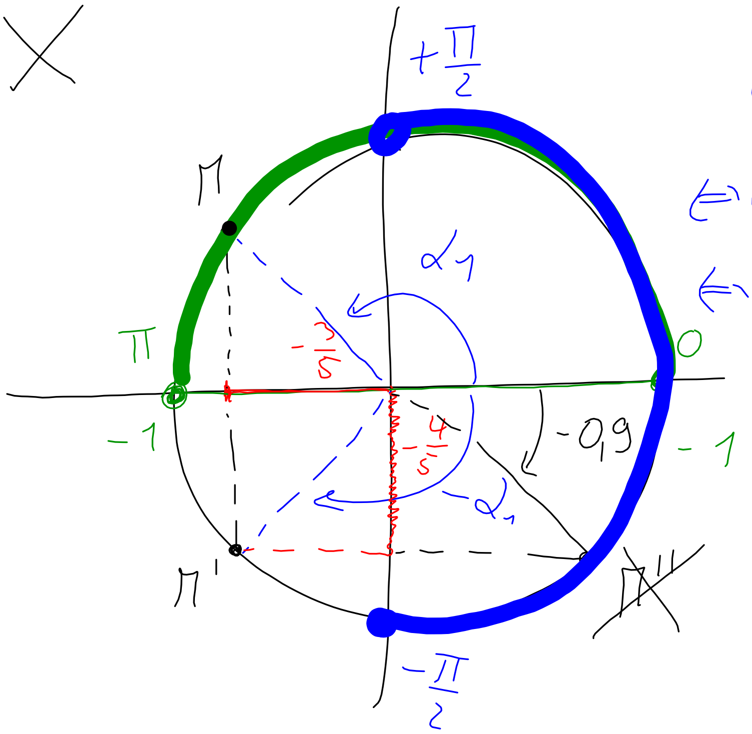
$$\text{et } \begin{cases} \alpha = \pi + k2\pi \\ \text{ou} \\ (\alpha = -\pi + k2\pi) \end{cases} \quad \text{et } \begin{cases} \cancel{\alpha = 0 + k2\pi} \\ \text{ou} \\ \alpha = \pi + k2\pi \end{cases}$$

$$\text{et } \alpha \in \{\pi + k2\pi\}$$

$$\text{ou } \cos(\alpha) = \frac{-3}{5} = \cos(-2{,}21) \text{ et } \sin(\alpha) = \frac{-4}{5} = \sin(-2{,}21)$$

$$\cancel{2{,}21} \qquad \cancel{-0{,}927}$$

$$\iff \alpha \in \{\pi + k2\pi \,;\, -2{,}21 + k2\pi\}$$



$$\cos(x) = -\frac{3}{5}$$

$$\Leftrightarrow \cos(x) \hat{=} \cos(2{,}21)$$

$$\Leftrightarrow \begin{cases} \cancel{x \hat{=} 2{,}21 + k2\pi} \\ \text{ou} \\ x \hat{=} -2{,}21 + k2\pi \end{cases}$$

h) $4\cos^2 t - 2(1+\sqrt{2})\cos t + \sqrt{2} = 0$

i) $\cos^2 x - 4\sin x\cos x + 35\cos^2 x = 0$

j) $\sin x\cos x + 2\cos^2 x = 0$

k) $5\sin^2 x + 35\sin x\cos x - 4 = 0$

l) $2\sin^2 x - \cos^2 x = \frac{1}{2} + 5\sin x\cos x$

m) $2\sin^4 x - \sin^2 x\cos^2 x - 3\cos^4 x = 0$

h) $4\cos^2(t) - 2(1+\sqrt{2})\cos(t) + \sqrt{2} = 0$

et $t \in \mathbb{R}$

$\Leftrightarrow \cos(t) = x$ et $4x^2 - 2(1+\sqrt{2})x + \sqrt{2} = 0$

$\Leftrightarrow \cos(t) = x$ et $4x^2 - 2x - 2\sqrt{2}x + \sqrt{2} = 0$

et $2x(2x-1) - \sqrt{2}(2x-1) = 0$

et $(2x-1)(2x-\sqrt{2}) = 0$

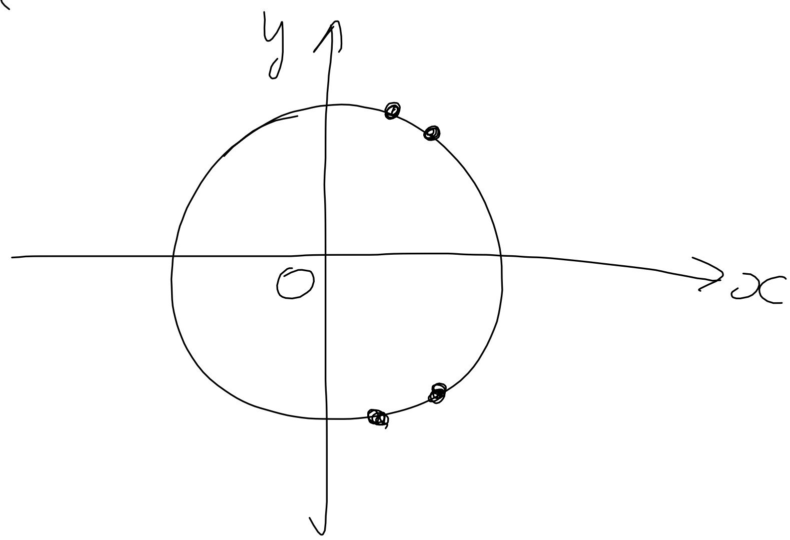
$\Leftrightarrow \cos(t) = \frac{1}{2}$ ou $\cos(t) = \frac{\sqrt{2}}{2}$

$\left\{\begin{array}{l} m+n = -2-2\sqrt{2} \\ m \cdot n = 4\sqrt{2} \end{array}\right.$

$\Leftrightarrow \left\{\begin{array}{l} m = -2 \\ n = -2\sqrt{2} \end{array}\right.$

$\Leftrightarrow \cos(t) = \cos\left(\frac{\pi}{3}\right)$ ou $\cos(t) = \cos\left(\frac{\pi}{4}\right)$

$\Leftrightarrow t \in \left\{ \frac{\pi}{3} + k2\pi \,;\, -\frac{\pi}{3} + k2\pi \,;\, \frac{\pi}{4} + k2\pi \,;\, -\frac{\pi}{4} + k2\pi \right\}$



$$4x^2-2(1+\sqrt{2})x+\sqrt{2}=0$$

$$\Leftrightarrow \Delta' = (+1+\sqrt{2})^2 - 4\cdot\sqrt{2}$$

$$= \underline{1+2\sqrt{2}+2} - 4\sqrt{2}$$

$$= 3-2\sqrt{2} = (1-\sqrt{2})^2 > 0$$

$$\text{et } x = \frac{(1+\sqrt{2}) \pm \sqrt{3-2\sqrt{2}}}{4}$$

$$= \frac{1+\sqrt{2} \pm \sqrt{(1-\sqrt{2})^2}}{4} = \frac{1+\sqrt{2} \pm (1-\sqrt{2})}{4}$$

$$\Leftrightarrow x = \frac{1+\sqrt{2}+1-\sqrt{2}}{4} \quad \text{ou} \quad x = \frac{1+\sqrt{2}-1+\sqrt{2}}{4}$$

$$\Leftrightarrow x = \frac{1}{2} \quad \text{ou} \quad x = \frac{\sqrt{2}}{2}$$

i) $\cos^2(x) - 4\sin(x)\cos(x) + 35\cos^2(x) = 0$

et $x \in \mathbb{R}$

$\iff \overset{9}{\cancel{36}}\cos^2(x) - \cancel{4}\sin(x)\cos(x) = 0$

$\iff \cos(x)\cdot\left(9\cos(x) - \sin(x)\right) = 0$

$\iff \cos(x) = 0$ ou $9\cos(x) - \sin(x) = 0$

$\iff x \in \left\{\frac{\pi}{2} + k\pi\right\}$ ou $x \in \left\{1{,}46 + k\pi\right\}$

$\iff x \in \left\{\frac{\pi}{2} + k\pi \,;\, 1{,}46 + k\pi\right\}$

$$9\cos(x) - \sin(x) = 0$$

$$\left(\Leftrightarrow \quad 9\cos(x) = \cancel{\sin x} \; \cos\left(\frac{\pi}{2} - x\right) \right)$$

?

$$\Leftrightarrow \quad 9 - \frac{\sin(x)}{\cos(x)} = 0 \qquad : (\cos(x) \neq 0)$$

$$\Leftrightarrow \quad 9 = \tan(x)$$

$$\Leftrightarrow \quad \tan(x) = 9 \simeq \tan(1{,}46)$$

$$\Leftrightarrow \quad x \in \{1{,}46 + k\pi\}$$

4) Résoudre les équations suivantes :
linéaires

a) $2\cos x + 3\sin x = 1$

b) $3\cos x + 2\sin x = -3$

c) $2\cos x + 3\sin x = 3$

d) $\cos x + 2\sin x = 4$

h) $4\cos^2 t - 2(1+\sqrt{2})\cos t + \sqrt{2} = 0$

i) $\cos^2 x - 4\sin x\cos x + 35\cos^2 x = 0$

j) $\sin x\cos x + 2\cos^2 x = 0$

k) $5\sin^2 x + 35\sin x\cos x - 4 = 0$

l) $2\sin^2 x - \cos^2 x = \frac{1}{2} + 5\sin x\cos x$

m) $2\sin^4 x - \sin^2 x\cos^2 x - 3\cos^4 x = 0$

Pièces jointes

- cercle trigo-1.fig
- Histoire du degre.pdf
- radian.fig
- cercle trigo-3.fig
- cercle trigo-2.fig
- enroulement-horiz-trigo.fig
- enroulement-horiz-trigo-rad.fig
- cercle trigo-sin-cos.fig
- cercle trigo-tan-cot.fig
- fct-paire-impaire.fig
- cercle trigo-1fig.fig