2 B - C sciences Mercredi 13 mai 2020

Examen de trigonométrie (2) - factice

Les formulaires et tables sont autorisées.
La calculatrice est autorisée, mais seulement pour les applications numériques en valeurs approchées.

Résoudre les équations trigonométriques suivantes :
(donner le domaine de chaque équation)

6 a) $\cos^2(x) = \frac{1}{2}$

8 b) $\sin(\frac{x}{2} - \frac{\pi}{3}) + \sin(\frac{x}{4} + \frac{2\pi}{3}) = 0$

10 c) $\sin(\alpha) - \cos(\alpha) = 1$

8 d) $1 + \sin(x) = \cos(2x)$

8 e) $\tan(2x) - 2\sin(x)\cos(x) = 0$

4 f) $3\sin(x) - 5\cos(x) = 7$

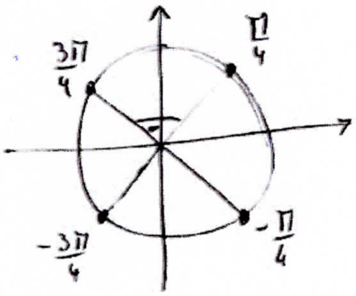
44 points

a) $\cos^2(x) = \frac{1}{2}$ et $x \in \mathbb{R}$ 1

$\Leftrightarrow \cos(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$ 1 $\Leftrightarrow \begin{cases} \cos(x) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right) & 1 \\ \text{ou} \\ \cos(x) = \frac{-\sqrt{2}}{2} = \cos\left(\frac{3\pi}{4}\right) & 1 \end{cases}$

6 $\Leftrightarrow x = \pm\frac{\pi}{4} + k2\pi$ ou $x = \pm 3\frac{\pi}{4} + k2\pi$ 1

$\Leftrightarrow x \in \left\{\frac{\pi}{4} + k\frac{\pi}{2}\right\}$ 1



b) $\sin\left(\frac{x}{2} - \frac{\pi}{3}\right) + \sin\left(\frac{x}{4} + \frac{2\pi}{3}\right) = 0$ et $x \in \mathbb{R}$ 1

1

$\Leftrightarrow \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) = -\sin\left(\frac{x}{4} + \frac{2\pi}{3}\right)$ 1

$\Leftrightarrow \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) = \sin\left(\pi + \frac{x}{4} + \frac{2\pi}{3}\right)$

$\Leftrightarrow \begin{cases} \frac{x}{2} - \frac{\pi}{3} = \frac{x}{4} + \frac{5\pi}{3} + k2\pi & 1 \\ \text{ou} \\ \frac{x}{2} - \frac{\pi}{3} = \pi - \left(\pi + \frac{x}{4} + \frac{2\pi}{3}\right) + k2\pi & 1 \end{cases}$

$\Leftrightarrow \begin{cases} \frac{x}{4} = \frac{6\pi}{3} + k2\pi & 1 \\ \text{ou} \\ \frac{3x}{4} = \frac{-\pi}{3} + k2\pi \end{cases} \Leftrightarrow \begin{cases} x = k8\pi & 1 \\ \text{ou} \\ x = \frac{-4\pi}{9} + k\frac{8\pi}{3} & 1 \end{cases}$

8 $\Leftrightarrow x \in \left\{k8\pi \,;\, \frac{-4\pi}{9} + k\frac{8\pi}{3}\right\}$

c) $\sin(\alpha) - \cos(\alpha) = 1$ et $\alpha \in \mathbb{R}$ 1

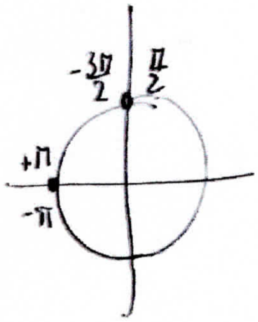
$\Leftrightarrow \frac{1}{\sqrt{2}}\sin(\alpha) - \frac{1}{\sqrt{2}}\cos(\alpha) = \underbrace{\frac{1}{\sqrt{2}}}_{\in [-1;1]}$ 1 $\quad) : \sqrt{2}$ 1

$\Leftrightarrow \sin\left(\frac{\pi}{4}\right)\sin(\alpha) - \cos\left(\frac{\pi}{4}\right)\cos(\alpha) = -\cos\left(\frac{\pi}{4}\right)$ 1 1

$\Leftrightarrow -\cos\left(\alpha + \frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$ 1

$\Leftrightarrow \cos\left(\pi + \alpha + \frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$ 1

$\Leftrightarrow \alpha + \frac{5\pi}{4} = \pm\frac{\pi}{4} + k2\pi$ 1 $\Leftrightarrow \begin{cases} \alpha = -\pi + k2\pi & 1 \\ \text{ou} \\ \alpha = -\frac{3\pi}{2} + k2\pi & 1 \end{cases}$



$\Leftrightarrow \alpha \in \left\{\pi + k2\pi \,;\, \frac{\pi}{2} + k2\pi\right\}$

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ou $\sin(\alpha) - \cos(\alpha) = 1$ et $\alpha \in \mathbb{R}$

$\Leftrightarrow \begin{cases} y - x = 1 & 1 \\ \text{et} \\ x^2 + y^2 = 1 & 1 \end{cases}$ et $x = \cos(\alpha)$ et $y = \sin(\alpha)$ 1

$y = x + 1$ et $x^2 + (x+1)^2 = 1$

$\Rightarrow 2x^2 + 2x = 0$

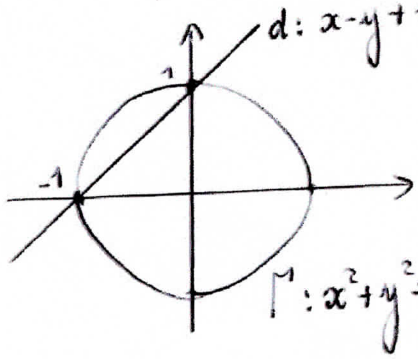
$\Leftrightarrow x(x+1) = 0$

$\Leftrightarrow x = 0$ ou $x = -1$

2

$\Leftrightarrow \begin{cases} x = \cos(\alpha) = 0 \text{ et } y = \sin(\alpha) = 1 & 1 \\ \text{ou} \\ x = \cos(\alpha) = -1 \text{ et } y = \sin(\alpha) = 0 & 1 \end{cases}$

$\Leftrightarrow \begin{cases} \alpha = \frac{\pi}{2} + k2\pi & 1 \\ \text{ou} \\ \alpha = \pi + k2\pi & 1 \end{cases}$



$\Leftrightarrow \alpha \in \left\{\frac{\pi}{2} + k2\pi \,;\, \pi + k2\pi\right\}$ 1

10

d) $1 + \sin(x) = \cos(2x)$ et $x \in \mathbb{R}$ 1

$\Leftrightarrow 1 + \sin(x) = \cos^2(x) - \sin^2(x)$ 1

$\Leftrightarrow \cancel{1} + \sin(x) = (\cancel{1} - \sin^2(x)) - \sin^2(x)$ 1

$\Leftrightarrow \sin(x) = -2\sin^2(x)$

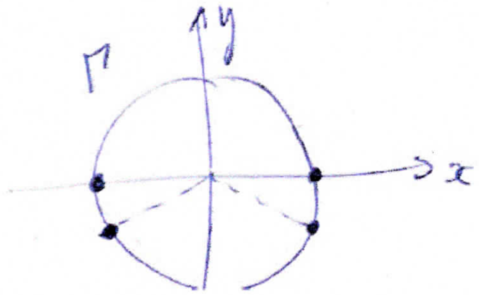
$\Leftrightarrow 2\sin^2(x) + \sin(x) = 0$ 1

$\Leftrightarrow \sin(x)\cdot(2\sin(x) + 1) = 0$ 1

$\Leftrightarrow \sin(x) = 0$ ou $\sin(x) = \frac{-1}{2} \left(= \sin\left(-\frac{\pi}{6}\right)\right)$ 1

$\Leftrightarrow x = k\pi$ 1 ou $\begin{cases} x = \frac{-\pi}{6} + k2\pi \\ \text{ou} \\ x = \pi - \left(\frac{-\pi}{6}\right) + k2\pi \end{cases}$ 1

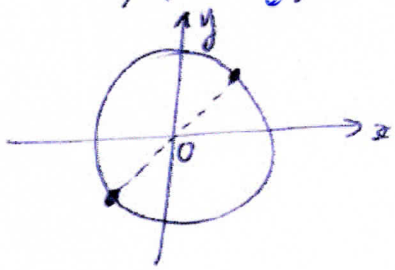
8 $\Leftrightarrow x \in \left\{ k\pi \,;\, \frac{-\pi}{6} + k2\pi \,;\, \frac{7\pi}{6} + k2\pi \mid k \in \mathbb{Z} \right\}$



e) $\tan(2x) - 2\sin(x)\cos(x) = 0$ et $x \in \mathbb{R} - \left\{ \frac{\pi}{4} + k\frac{\pi}{2} \right\}$ 1

$\Leftrightarrow \frac{\sin(2x)}{\cos(2x)} - \sin(2x) = 0$ 1 1

$\Leftrightarrow \sin(2x)\cdot\left(\frac{1}{\cos(2x)} - 1\right) = 0$ 1



$\Leftrightarrow \sin(2x) = 0$ ou $\frac{1}{\cos(2x)} = 1$

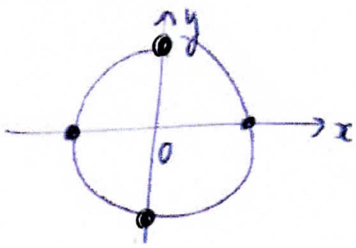
$\Leftrightarrow \sin(2x) = \sin(0)$ ou $\cos(2x) = 1 = \cos(0)$ 1

$\Leftrightarrow 2x = 0 + k\pi$ ou $2x = 0 + k2\pi$ 1 1

$\Leftrightarrow 2x = k\pi$

$\Leftrightarrow x = k\frac{\pi}{2}$ 1

$\Leftrightarrow x \in \left\{ k\frac{\pi}{2} \mid k \in \mathbb{Z} \right\}$



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f) $3\sin(x) - 5\cos(x) = 7$ et $x \in \mathbb{R}$ 1

$\Leftrightarrow \frac{3}{\sqrt{3^2+(-5)^2}}\sin(x) - \frac{5}{\sqrt{3^2+(-5)^2}}\cos(x) = \frac{7}{\sqrt{3^2+(-5)^2}}$ 2

$\Leftrightarrow x \in \varnothing$

4 car $\frac{7}{\sqrt{34}} = \frac{\sqrt{49}}{\sqrt{34}} = \sqrt{\frac{49}{34}} > 1$ 1