* $\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$ (car "cos" est paire)

$= \frac{1}{2}$ (cf Tableau)

* $\sin\left(\frac{3\pi}{2}\right) = -1$ (cf Tableau)

* $\cos\left(\frac{5\pi}{2}\right) = \cos\left(\frac{4\pi}{2} + \frac{\pi}{2}\right) = \cos\left(2\pi + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right)$ ("cos" est pér. de 2π)

$= 0$

* $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ (cf tableau)

* $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ (cf tableau)

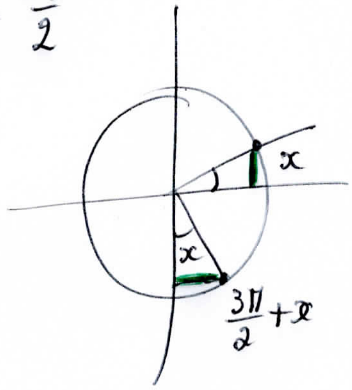
* $\sin\left(36\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

* $\cos\left(-12\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

* $\sin\left(3\pi + \frac{\pi}{3}\right) = \sin\left(2\pi + \left(\pi + \frac{\pi}{3}\right)\right) = \sin\left(\frac{4\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

* $\cos\left(5\pi - \frac{\pi}{3}\right) = \cos\left(4\pi + \left(\pi - \frac{\pi}{3}\right)\right) = \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$

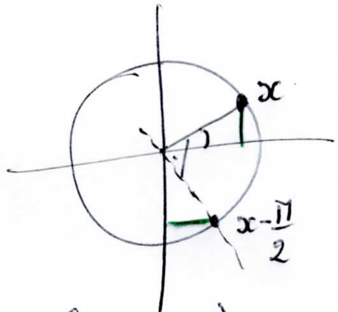
* $\cos\left(x + \frac{3\pi}{2}\right) = \sin(x)$



* $\cos\left(x - \frac{5\pi}{2}\right) = \cos\left(x - \frac{\pi}{2} - 2\pi\right)$

$= \cos\left(x - \frac{\pi}{2}\right)$

$= \sin(x)$



* $\sin\left(145\frac{\pi}{6}\right) = \sin\left(\frac{144\pi}{6} + \frac{\pi}{6}\right) = \sin\left(24\pi + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

* $\sin\left(\frac{218\pi}{3}\right) = \sin\left(\frac{216\pi}{3} + \frac{2\pi}{3}\right) = \sin\left(72\pi + \frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

* $\cos\left(-\frac{15\pi}{4}\right) = \cos\left(\frac{15\pi}{4}\right) = \cos\left(\frac{16\pi}{4} - \frac{\pi}{4}\right) = \cos\left(4\pi - \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) =$
$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

* $\sin\left(\frac{73\pi}{6}\right) = \sin\left(\frac{72\pi}{6} + \frac{\pi}{6}\right) = \sin\left(12\pi + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

* $\sin(x - 3\pi) = \sin(x - \pi - 2\pi) = \sin(x - \pi) = -\sin(\pi - x) \overset{*}{=} -\sin(x)$

* $\cos(5\pi + x) = \cos(4\pi + \pi + x) = \cos(\pi + x) \overset{*}{=} -\cos(x)$

* $\cos(4\pi - x) = \cos(-x) = \cos(x)$ (car "cos" est paire)

* $\cos\left(\frac{5\pi}{2} - x\right) = \cos\left(\frac{4\pi}{2} + \frac{\pi}{2} - x\right) = \cos\left(\frac{\pi}{2} - x\right) \overset{*}{=} \sin(x)$

* $\sin\left(\frac{7\pi}{2} - x\right) = \sin\left(\frac{8\pi}{2} - \frac{\pi}{2} - x\right) = \sin\left(-\frac{\pi}{2} - x\right) = \sin\left(\frac{\pi}{2} + x\right) \overset{*}{=} \cos(x)$

car "sin" est impaire

* $\cos\left(\frac{7\pi}{2} - x\right) = \cos\left(\frac{8\pi}{2} - \frac{\pi}{2} - x\right) = \cos\left(-\frac{\pi}{2} - x\right)$ car "cos" est paire

$= \cos\left(\frac{\pi}{2} + x\right)$

$\overset{*}{=} -\sin(x)$

* cf Tableau du début de ce cours (cf Form. et Tables)

exercice 29

a) Calculer $\sin(x)$, si $\tan(x) = \frac{3}{2}$ et $2\sin(x) + 3\cos(x) = 2$
et $\cos(x)$
(sans calculer x)

$*$ Posons $\sin(x) = t$ et $\cos(x) = u$

alors $\tan(x) = \frac{3}{2} \Leftrightarrow \frac{\sin(x)}{\cos(x)} = \frac{t}{u} = \frac{3}{2} \Leftrightarrow 2t - 3u = 0$

et $2\sin(x) + 3\cos(x) = 2 \Leftrightarrow 2t + 3u =$

à résoudre : $\begin{cases} 2t - 3u = 0 \\ 2t + 3u = 2 \end{cases} \Leftrightarrow \begin{cases} 4t = 2 \text{ et } t = \frac{1}{2} \\ 6u = 2 \text{ et } u = \frac{1}{3} \end{cases}$

Réponse : $t = \sin(x) = \frac{1}{2}$ et $u = \cos(x) = \frac{1}{3}$

b) Sans calculer x, calculer $\sin(x)$ et $\cos(x)$ si $\tan(x) = \frac{3}{4}$

$*$ Posons $\sin(x) = t$ et $\cos(x) = u$ d'où $\tan(x) = \frac{t}{u} = \frac{3}{4}$

$\Leftrightarrow 4t - 3u = 0$ ... mais il nous manque une équation !

Or $\sin^2(x) + \cos^2(x) = 1, \forall x \in \mathbb{R} \Rightarrow t^2 + u^2 = 1$

à résoudre : $\begin{cases} 4t - 3u = 0 \\ t^2 + u^2 = 1 \end{cases} \Leftrightarrow \begin{cases} t = \frac{3}{4}u \\ \left(\frac{3}{4}u\right)^2 + u^2 = 1 \end{cases} \Leftrightarrow \begin{cases} t = \pm\frac{3}{5} \\ u = \pm\frac{4}{5} \end{cases}$

réponse : $\sin(x) = t = \frac{3}{5}$
et
$\cos(x) = u = \frac{4}{5}$

ou $\sin(x) = \frac{-3}{5}$
et
$\cos(x) = \frac{-4}{5}$

coin brico : $\left(\frac{3}{4}u\right)^2 + u^2 = 1$

$\Leftrightarrow \frac{9}{16}u^2 + \frac{16}{16}u^2 = 1$

$\Leftrightarrow \frac{25}{16}u^2 = 1 \Leftrightarrow u^2 = \frac{16}{25} \Leftrightarrow u = \pm\frac{4}{5}$

d'où $t = \frac{3}{4}u = \pm\frac{3}{4}\cdot\frac{4}{5} = \pm\frac{3}{5}$