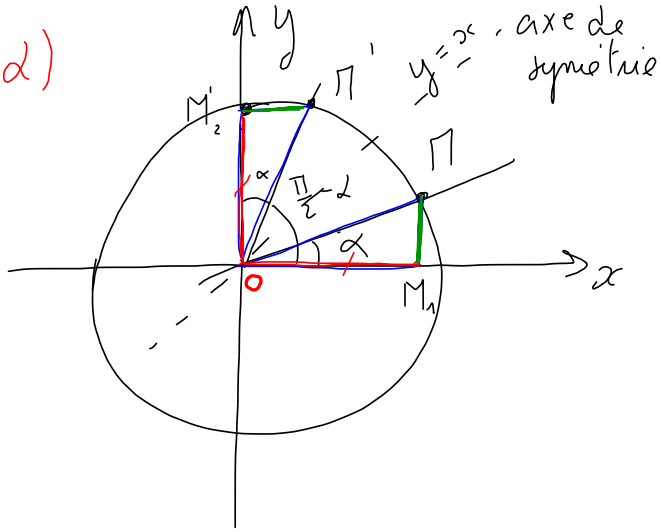
Relations entre fonctions trigonométriques de certains arcs

$\cos(-\alpha) = \cos(\alpha)$	$\sin(-\alpha) = -\sin(\alpha)$	$\tan(-\alpha) = -\tan(\alpha)$
$\cos(\pi - \alpha) = -\cos(\alpha)$	$\sin(\pi - \alpha) = \sin(\alpha)$	$\tan(\pi - \alpha) = -\tan(\alpha)$
$\cos(\pi + \alpha) = -\cos(\alpha)$	$\sin(\pi + \alpha) = -\sin(\alpha)$	$\tan(\pi + \alpha) = \tan(\alpha)$
$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$	$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos(\alpha)$	$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot(\alpha)$
$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin(\alpha)$	$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos(\alpha)$	$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot(\alpha)$

$*$ $\sin\left(\frac{\pi}{2}-\alpha\right) = \cos(\alpha)$

$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin(\alpha)$

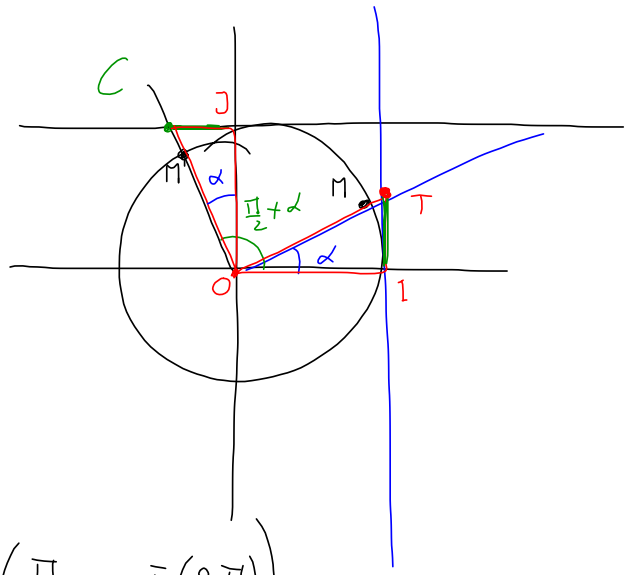


$* \cot\left(\frac{\pi}{2}+\alpha\right) =$

$* \cos\left(\frac{31\pi}{3}\right) =$

$* \cos\left(\frac{31\pi}{2}-\alpha\right) =$

$* \cot\left(\frac{\pi}{2}+\alpha\right) = -\tan(\alpha)$

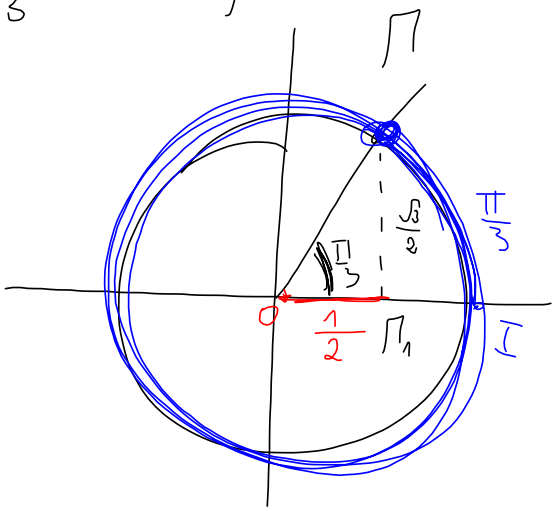


$* \cos\left(\frac{31\pi}{3}\right) =$

$\cos\left(\frac{30\pi}{3} + \frac{\pi}{3}\right) =$

$\cos\left(10\pi + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} + 5(2\pi)\right)$

$= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

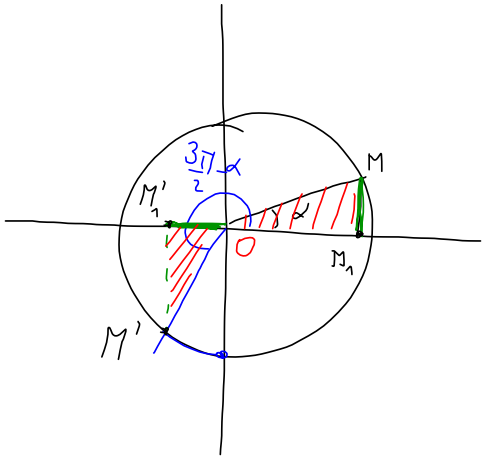


$* \cos\left(\frac{31\pi}{2} - \alpha\right) =$

$\cos\left(\frac{28\pi}{2} + \frac{3\pi}{2} - \alpha\right) =$

$\cos\left(\frac{3\pi}{2} - \alpha\right) =$

$-\sin(\alpha)$

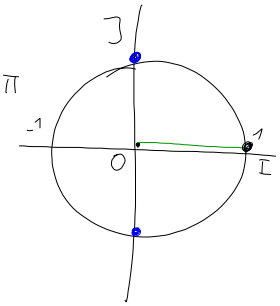


§ 3 Les équations trigonométriques élémentaires

1) $\cos(x) = 0$ et $x \in \mathbb{R}$

$\Leftrightarrow x = \frac{\pi}{2} + k2\pi$ ou $x = \frac{3\pi}{2} + k2\pi$

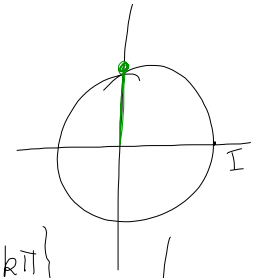
$\Leftrightarrow x \in \left\{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}$



2) $\cos(x) = 1$ et $x \in \mathbb{R}$

$\Leftrightarrow x = 0 + k2\pi$

$\Leftrightarrow x \in \{ k2\pi \mid k \in \mathbb{Z} \}$

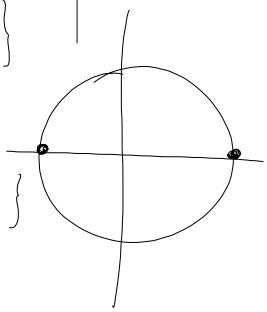


3) $\sin(x) = 1$ et $x \in \mathbb{R}$

$\Leftrightarrow x \in \left\{ \frac{\pi}{2} + k2\pi \right\}$

4) $\tan(x) = 0$ et $x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \right\}$

$\Leftrightarrow x = 0 + k\pi \Leftrightarrow x \in \{ k\pi \}$



5) $\tan(x) = 1$ et $x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \right\}$

$\Leftrightarrow x \in \left\{ \frac{\pi}{4} + k\pi \right\}$

6) $\cot(x) = 0$ et $x \in \mathbb{R} - \{ k\pi \}$

$\Leftrightarrow x \in \left\{ \frac{\pi}{2} + k\pi \right\}$

7) $\cos(x) = 3$ et $x \in \mathbb{R}$

$\Leftrightarrow x \in \emptyset$ ($\cos 3 \notin [-1; 1]$)

8) $\tan(x) = 3$ et $x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \right\} \Leftrightarrow x =$??

9) $\tan(x) = \frac{\sqrt{3}}{3} \Leftrightarrow x \in \{$ $\}$

10) Soit $a \in \mathbb{R}$:

* $\tan(x) = a \Leftrightarrow$

$\Leftrightarrow x \in \{$

* $\cot(x) = a \Leftrightarrow$

$\Leftrightarrow x \in \{$

11) $\cos(x) = a \Leftrightarrow$

$\sin(x) = a \Leftrightarrow$

1) Soit $a \in \mathbb{R}$: $\tan(x) = a$ ↘ avec le tableau ou avec la calculatrice

$\Leftrightarrow \tan(x) = \tan(\alpha)$

$\Leftrightarrow x \in \{\alpha + k\pi\}$

et $\cot(x) = a$ ↘ idem

$\Leftrightarrow \cot(x) = \cot(\beta)$

$\Leftrightarrow x \in \{\beta + k\pi\}$

2) Soit $a \in [-1;1]$ et $\cos(x) = a$ ↘ avec le tableau ou la calculatrice

$\Leftrightarrow \cos(x) = \cos(\alpha)$

$\Leftrightarrow \begin{cases} x = \alpha + k \cdot 2\pi \\ \text{ou} \\ x = -\alpha + k \cdot 2\pi \end{cases}$ $\Leftrightarrow x \in \{\alpha + k2\pi ; -\alpha + k2\pi\}$

3) Soit $a \in [-1;1]$: $\sin(x) = a$ ↘ avec le tableau ou avec la calculatrice

$x = \pi - \alpha$

$\Leftrightarrow x + \alpha = \pi$ $\Leftarrow \sin(x) = \sin(\alpha)$

$\Leftarrow \begin{cases} x = \alpha + k \cdot 2\pi \\ \text{ou} \\ x = (\pi - \alpha) + k \cdot 2\pi \end{cases}$ $\Leftarrow x \in \{\alpha + k \cdot 2\pi ; (\pi - \alpha) + k \cdot 2\pi\}$

20 *Calculer* $\cos(-\frac{\pi}{3})$ $\sin\frac{3\pi}{2}$ $\cos\frac{5\pi}{2}$ $\sin\frac{5\pi}{4}$ $\cos\frac{7\pi}{6}$

$\sin(36\pi+\frac{\pi}{3})$ $\cos(-12\pi+\frac{\pi}{6})$ $\sin(3\pi+\frac{\pi}{3})$ $\cos(5\pi-\frac{\pi}{3})$ $\cos(x+\frac{3\pi}{2})$

$\cos(x-\frac{5\pi}{2})$ $\sin\frac{145\pi}{6}$ $\sin\frac{218\pi}{3}$ $\cos(-\frac{15\pi}{4})$ $\sin\frac{73\pi}{6}$

23 *Exprimer en fonction de* $\cos x$ *ou de* $\sin x$.

$\sin(x-3\pi)$ $\cos(5\pi+x)$ $\cos(4\pi-x)$ $\cos(\frac{5\pi}{2}-x)$ $\sin(\frac{7\pi}{2}-x)$ $\cos(\frac{7\pi}{2}-x)$

29 *a)* *Sans calculer* x, *trouver* $\sin x$ *et* $\cos x$ *si* $\tan x = \frac{3}{2}$ *et* $2\sin x + 3\cos x = 2$.

b) *Sans calculer* x, *trouver* $\sin x$ *et* $\cos x$ *si* $\tan x = \frac{3}{4}$.

Pièces jointes

- cercle trigo-1.fig
- Histoire du degre.pdf
- radian.fig
- cercle trigo-3.fig
- cercle trigo-2.fig
- enroulement-horiz-trigo.fig
- enroulement-horiz-trigo-rad.fig
- cercle trigo-sin-cos.fig
- cercle trigo-tan-cot.fig
- fct-paire-impaire.fig