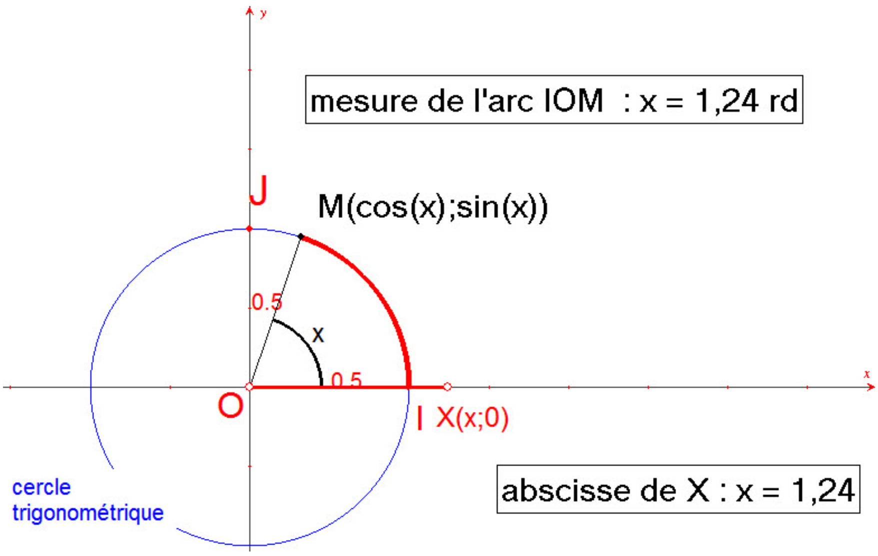
enroulement-horiz-trigo



Relations entre fonctions trigonométriques d'un même arc

$\cos^2(\alpha) + \sin^2(\alpha) = 1$	$\tan(\alpha) = \dfrac{\sin(\alpha)}{\cos(\alpha)}$	$\cot(\alpha) = \dfrac{\cos(\alpha)}{\sin(\alpha)}$
$\cot(\alpha) = \dfrac{1}{\tan(\alpha)}$	$\dfrac{1}{\cos^2(\alpha)} = 1 + \tan^2(\alpha)$	$\dfrac{1}{\sin^2(\alpha)} = 1 + \cot^2(\alpha)$

Formules trigo : cf Formulaires p. 29 à 32

1) $\cos^2(x) + \sin^2(x) = 1$, $\forall x \in \mathbb{R}$

$\left(\begin{array}{l} \Leftrightarrow \sin^2(x) = 1 - \cos^2(x) \\ \Leftrightarrow \cos^2(x) = 1 - \sin^2(x) \end{array} \right)$

2) $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, $\forall x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \right\}$

3) $\cot(x) = \dfrac{1}{\tan(x)}$, $\forall x \in \mathbb{R} - \{ k\pi \}$

4) $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$, $\forall x \in \mathbb{R} - \{ k\pi \}$

5) $\dfrac{1}{\sin^2(x)} = 1 + \cot^2(x)$, $\forall x \in \mathbb{R} - \{ k\pi \}$

6) $\dfrac{1}{\cos^2(x)} = 1 + \tan^2(x)$, $\forall x \in \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi \right\}$

preuve de 6)

tangente

H $x \in \mathbb{R} - \{ \frac{\pi}{2} + k\pi \}$

T $\frac{1}{\cos^2(x)} = 1 + \tan^2(x)$

D $\frac{1}{\cos^2(x)} \overset{①}{=} \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)}$

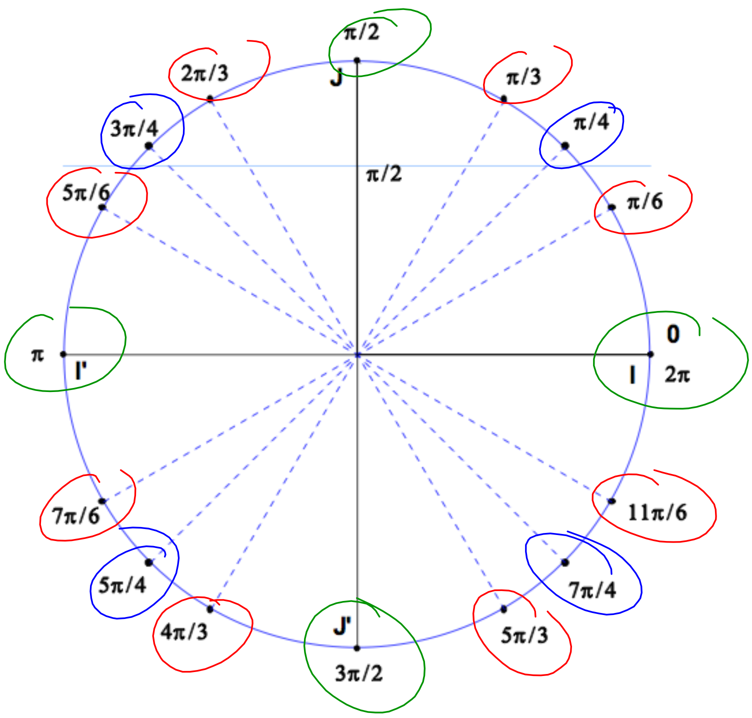
algèbre

algèbre

$= 1 + \left(\frac{\sin(x)}{\cos(x)} \right)^2 \overset{②}{=} 1 + \tan^2(x)$ cqfd

§ 4 Calculs des valeurs trigonométriques de référence : fig

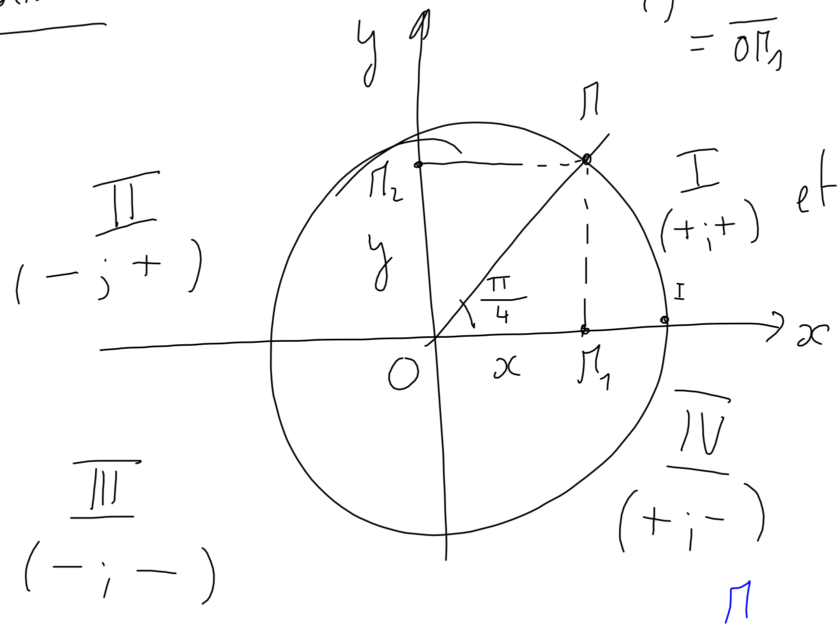
x (radian)	cos (x)	sin (x)	tan (x)	cot (x)
0	1	0	0	X
π/6				
π/4				
π/3				
π/2	0	1	X	0
2π/3				
3π/4				
5π/6				
π	-1	0	0	X
7π/6				
5π/4				
4π/3				
3π/2	0	-1	X	0
5π/3				
7π/4				
11π/6				
2π	1	0	0	X


π/2
2π/3
π/3
J
3π/4
π/4
π/2
5π/6
π/6
0
π
I'
I
2π
7π/6
11π/6
5π/4
7π/4
4π/3
J'
5π/3
3π/2

§ 4 Calculs des valeurs trigonométriques de référence :

x (radian)	cos (x)	sin (x)	tan (x)	cot (x)
o	1	0	0	X
$\pi/6$				
$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\pi/3$				
$\pi/2$	0	1	X	0
$2\pi/3$				
$3\pi/4$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1
$5\pi/6$				
π	-1	0	0	X
$7\pi/6$				
$5\pi/4$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1
$4\pi/3$				
$3\pi/2$	0	-1	X	0
$5\pi/3$				
$7\pi/4$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1
$11\pi/6$				
2π	1	0	0	X

Cas limites : Calcul de $\cos\left(\frac{\pi}{4}\right) = x = \overline{OM_1}$ et $\sin\left(\frac{\pi}{4}\right) = y = \overline{OM_2}$



et $x > 0$ et $y > 0$

On a : le triangle OM_1M est isocèle :

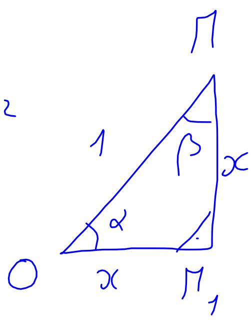
car : $\sphericalangle OM_1M = \sphericalangle d$

$\sphericalangle MOM_1 = \sphericalangle \alpha$

et $\alpha = 45°$

et donc $\beta = 45°$

si $\sphericalangle \beta = \sphericalangle M_1MO$



Donc $OM_1 = x = y = M_1M = OM_2$

et par Pythagore :

$$OM^2 = OM_1^2 + M_1M^2$$

$\Leftrightarrow 1 = x^2 + x^2$

$\Leftrightarrow 1 = 2x^2$ | $\Leftrightarrow 2x^2 - 1 = 0$

$\Leftrightarrow x^2 = \frac{1}{2}$ | $\Leftrightarrow (\quad)(\quad) = 0$

$\Leftrightarrow x = +\cancel{-}\sqrt{\frac{1}{2}} = +\cancel{-}\frac{1}{\sqrt{2}} = +\cancel{-}\frac{\sqrt{2}}{2}$ car $x > 0$

§ 4 Calculs des valeurs trigonométriques de référence :

x (radian)	cos (x)	sin (x)	tan (x)	cot (x)
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5π/6				
π	−1	0	0	X
7π/6				
5π/4	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	1
4π/3				
3π/2	0	−1	X	0
5π/3				
7π/4	$\sqrt{2}/2$	$-\sqrt{2}/2$	−1	−1
11π/6				
2π	1	0	0	X

§ 4 Calculs des valeurs trigonométriques de référence :

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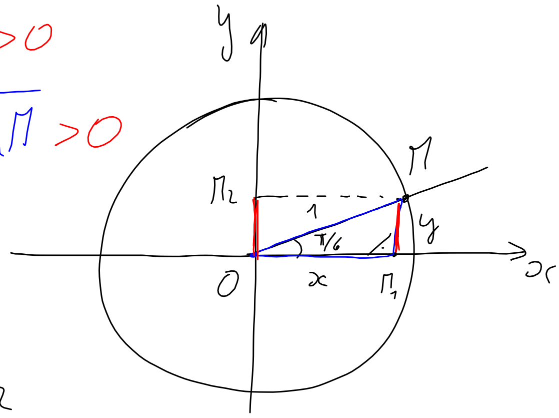
3π/4	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	-1
5π/6				
π	-1	0	0	X
7π/6				
5π/4	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	1
4π/3				
3π/2	0	-1	X	0
5π/3				
7π/4	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	-1
11π/6				
2π	1	0	0	X

* Calculs des valeurs trigonométrique en $\frac{\pi}{6}$ (30°)



Soit $\cos\left(\frac{\pi}{6}\right) = x = \overline{OM_1}$ >0

$\sin\left(\frac{\pi}{6}\right) = y = \overline{OM_2} = \overline{M_1M} > 0$

Soit le triangle ΔOM_1M

rectangle en M_1

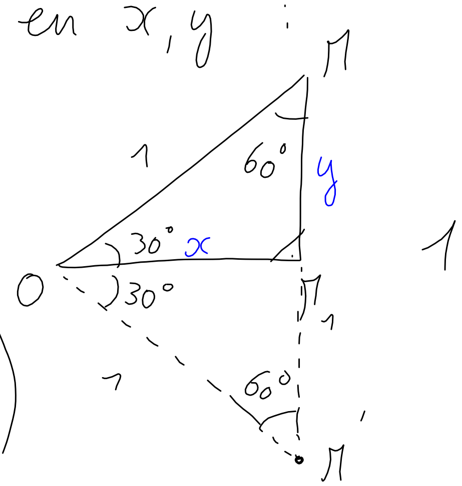
et par Pythagore : $x^2 + y^2 = 1$

Il nous faut une 2e équation en "x, y" :

Soit $M' = S_{M_1}(M)$

et le triangle $\Delta OM'M$

est alors équilatéral (3 angles à 60°)



donc $OM = MM' = M'O = 1$

et aussi $y = M_1M = \frac{1}{2} MM' = \frac{1}{2}$

et alors : $x^2 + y^2 = 1$ et $y = \frac{1}{2} \Longrightarrow x^2 + \frac{1}{4} = 1$

$$\Leftrightarrow x^2 = \frac{3}{4} \Leftrightarrow x = \cancel{\pm}\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

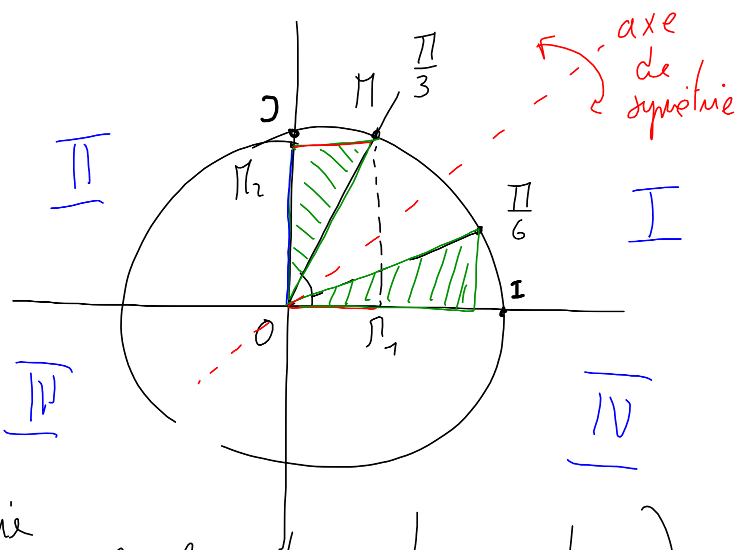
de plus : $\tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} : \frac{\sqrt{3}}{2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

et $\cot\left(\frac{\pi}{6}\right) = \frac{1}{\tan\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

* pour $\frac{\pi}{3}$:

$\cos\left(\frac{\pi}{3}\right) = \overline{OM_1} = \overline{M_2 M}$

$\sin\left(\frac{\pi}{3}\right) = \overline{OM_2}$



Ces triangles sont isométriques (symétrie d'axe la bissectrice des quadrants I et III d'équation $y = x$)

d'où : $\cos\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

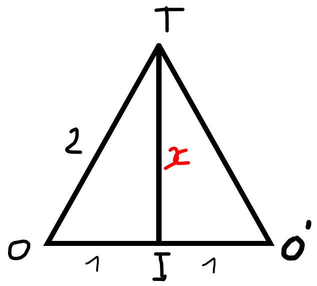
$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

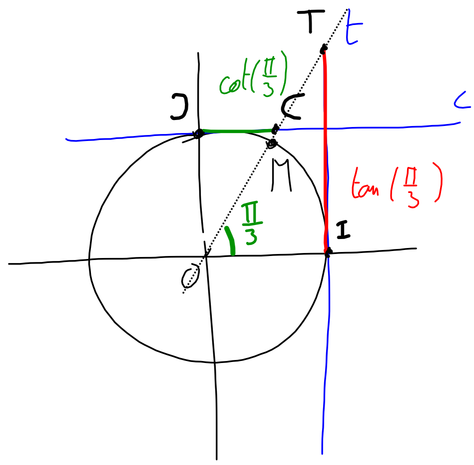
et $\tan\left(\frac{\pi}{3}\right) = \dfrac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \sqrt{3}$ et $\cot\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

Calculer $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\cot\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

$4 = 1 + x^2$





Calculs des valeurs trigonométriques de référence :

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$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1
$\pi/3$	$\frac{1}{2}$	$\sqrt{3}/2$	$\sqrt{3}$	$\sqrt{3}/3$
$\pi/2$	0	1	X	0
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$3\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	-1
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2π	1	0	0	X

Pièces jointes

- cercle trigo-1.fig
- Histoire du degre.pdf
- radian.fig
- cercle trigo-3.fig
- cercle trigo-2.fig
- enroulement-horiz-trigo.fig
- enroulement-horiz-trigo-rad.fig
- cercle trigo-tan-cot.fig