

## Exercices de mathématique — Série I

( Factorisations )

1) Vérifier que le nombre  $-3$  est racine de la fonction  $f$  définie par  $f(x) = 2x^3 + 9x^2 + 7x - 6$ , puis factoriser  $f(x)$ .

2) Calculer les racines de  $f$  : (  $\Leftrightarrow$  résoudre  $f(x) = 0$  )

a)  $f(x) = x^4 - 2x^3 - 9x^2 + 2x + 8$

b)  $f(x) = (3 + 7x)(4 - x^2) \left(\frac{3}{5}x - 2\right)$

c)  $f(x) = (x-1)^3 - 27$

d)  $f(x) = (2x + 1)(2x + 3) - 30x - 15$

e)  $f(x) = x^3 - 13x + 12$

f)  $f(x) = 12x^2 - 2x - 2$

g)  $f(x) = x^4 + 3x^2 - 4$

h)  $f(x) = x^2 - 4x - 77$

$$a) \quad f(x) = x^4 - 2x^3 - 9x^2 + 2x + 8 = 0$$

$$\Leftrightarrow (x-1)(x^3 - x^2 - 10x - 8) = 0$$

$$\Leftrightarrow (x-1)(x+1) \overset{g(x)}{(x^2 - 2x - 8)} = 0$$

$$\Leftrightarrow (x-1)(x+1)(x-4)(x+2) = 0$$

$$\Leftrightarrow x \in \{1; -1; 4; -2\}$$

$$f(1) = 1 - 2 - 9 + 2 + 8 = 0$$

$$\begin{array}{ccc|c} 1 & -2 & -9+2 & +8 \\ & & 1 & -1-10 & -8 \\ \hline 1 & 1 & -1 & -10 & -8 & 0 \end{array}$$

$$g(1) = 1 - 1 - 10 - 8 \neq 0$$

$$g(-1) = -1 - 1 + 10 - 8 = 0$$

$$\begin{array}{ccc|c} 1 & -1 & -10 & -8 \\ & & -1 & +2 & +8 \\ \hline -1 & 1 & -2 & -8 & 0 \end{array}$$

$$b) \quad f(x) = (3+7x)(4-x^2)\left(\frac{3}{5}x-2\right) = 0$$

$$\Leftrightarrow (7x+3)(2-x)(2+x)\left(\frac{3}{5}x-2\right) = 0$$

$$\Leftrightarrow x \in \left\{-\frac{3}{7}; 2; -2; \frac{10}{3}\right\}$$

$$\frac{3}{5}x - 2 = 0$$

$$\Leftrightarrow \frac{3}{5}x = 2$$

$$\Leftrightarrow x = 2 \cdot \frac{5}{3} = \frac{10}{3}$$

$$d) \quad f(x) = (2x+1)(2x+3) - 30x - 15 = 0$$

$$\Leftrightarrow (2x+1)(2x+3) - 15(2x+1) = 0$$

$$\Leftrightarrow (2x+1)[(2x+3) - 15] = 0$$

$$\Leftrightarrow (2x+1)(2x-12) = 0$$

$$\Leftrightarrow (2x+1) \cdot 2 \cdot (x-6) = 0$$

$$\Leftrightarrow x \in \left\{-\frac{1}{2}; 6\right\}$$

$$e) \quad f(x) = x^3 - 13x + 12 = 0$$

$$\Leftrightarrow (x-1)(x^2+x-12) = 0$$

$$\Leftrightarrow (x-1)(x+4)(x-3) = 0$$

$$\Leftrightarrow x \in \{1; -4; 3\}$$

$$f(1) = 1 - 13 + 12 = 0$$

$$\begin{array}{ccc|c} 1 & 0 & -13 & 12 \\ & & 1 & 1 & -12 \\ \hline 1 & 1 & 1 & -12 & 0 \end{array}$$

$$f) \quad f(x) = 12x^2 - 2x - 2 = 0$$

$$\Leftrightarrow 6x^2 - x - 1 = 0$$

$$\begin{cases} m+n = -1 \\ m \cdot n = -6 \end{cases}$$

$$\Leftrightarrow 6x^2 - 3x + 2x - 1 = 0$$

$$\Leftrightarrow 3x(2x-1) + (2x-1) = 0$$

$$\Leftrightarrow (2x-1)(3x+1) = 0$$

$$\Leftrightarrow x \in \left\{ \frac{1}{2}; -\frac{1}{3} \right\}$$

$$\Leftrightarrow \begin{cases} m = -3 \\ m = +2 \end{cases}$$

$$g) \quad f(x) = x^4 + 3x^2 - 4 = 0$$

$$\Leftrightarrow y = x^2 \text{ et } y^2 + 3y - 4 = 0$$

$$\Leftrightarrow y = x^2 \text{ et } (y+4)(y-1) = 0$$

$$\Leftrightarrow (x^2+4)(x^2-1) = 0$$

$$\Leftrightarrow x^2+4=0 \text{ ou } x^2-1=0$$

$$\Leftrightarrow x \in \emptyset \text{ ou } (x-1)(x+1) = 0$$

$$\Leftrightarrow x \in \{1; -1\}$$

$$h) \quad f(x) = x^2 - 4x - 77 = 0$$

$$\Leftrightarrow (x-11)(x+7) = 0$$

$$\Leftrightarrow x \in \{11; -7\}$$

$$\begin{cases} m+n = -4 \\ m \cdot n = -77 \end{cases}$$

$$\Leftrightarrow \begin{cases} m = -11 \\ m = +7 \end{cases}$$